

Numerical Method for Solving Obstacle Scattering Problems by an Algorithm Based on the Modified Rayleigh Conjecture

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Abstract. In this paper we present a numerical algorithm for solving the direct scattering problems by the Modified Rayleigh Conjecture Method (MRC) introduced in [1]. Some numerical examples are given. They show that the method is numerically efficient.

Key words. direct obstacle scattering problem, Modified Rayleigh Conjecture, MRC algorithm

AMS Subject Classification. 65Z05, 35R30

I. Introduction

The classical Rayleigh Conjecture is discussed in [4] and [5], where it is shown that, in general, this conjecture is incorrect: there are obstacles (for example, sufficiently elongated ellipsoids) for which the series, representing the scattered field outside a ball containing the obstacle, does not converge up to the boundary of this obstacle.

The Modified Rayleigh Conjecture (MRC) has been formulated and proved in [1] (see Theorem 1 below). A numerical method for solving obstacle scattering problems, based on MRC, was proposed in [1]. This method was implemented in [2] for two-dimensional obstacle scattering problems. The numerical results in [2] were quite encouraging: they show that the method is efficient, economical, and is quite competitive compared with the usual boundary integral equations method (BIEM). A recent paper [3] contains a numerical implementation of MRC in some three-dimensional obstacle scattering problems. Its results reconfirm the practical efficiency of the MRC method.

In this paper a numerical implementation of the Modified Rayleigh Conjecture (MRC) method for solving obstacle scattering problem in three-dimensional case is presented. Our aim is to consider more general than in [3] three-dimensional obstacles: non-convex, non-starshaped, non-smooth, and to study the performance of the MRC in these cases. The minimization problem (5) (see below), which is at the heart of the MRC method, is treated numerically in a new way, different from the one used in [2] and [3]. Our results present further numerical evidence of the practical efficiency of the MRC method for solving obstacle scattering problems.

The obstacle scattering problems (1)-(3), we are interested in, consists

of solving the equation

$$(\nabla^2 + k^2)u = 0 \quad \text{in } D' = \mathbb{R}^3 \setminus D, \quad (1)$$

where $D \subset \mathbb{R}^3$ is a bounded domain, satisfies the Dirichlet boundary condition

$$u|_S = 0, \quad (2)$$

where S is the boundary of D , which is assumed Lipschitz in this paper, and the radiation condition at infinity:

$$u = u_0 + v = u_0 + A(\alpha', \alpha) \frac{e^{ikr}}{r} + o\left(\frac{1}{r}\right) \quad r \rightarrow \infty, \quad (3)$$

$$r := |x|, \quad \alpha' = x/r, \quad u_0 := e^{ik\alpha \cdot x},$$

where v is the scattered field, $\alpha \in S^2$ is given, S^2 is the unit sphere in \mathbb{R}^3 , $k = const > 0$ is fixed, k is the wave number. The coefficient $A(\alpha', \alpha)$ is called the scattering amplitude.

Denote

$$A_l(\alpha) := \int_{S^2} A(\alpha', \alpha) \overline{Y_l(\alpha')} d\alpha', \quad (4)$$

where $Y_l(\alpha)$ are the orthonormal spherical harmonics:

$$Y_l = Y_{lm}, \quad -l \leq m \leq l, \quad l = 0, 1, 2, \dots$$

$$Y_{lm}(\theta, \phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \Theta_{lm}(\cos\theta),$$

$$\Theta_{lm}(x) = \sqrt{\frac{2l+1}{2} \frac{(l-m)!}{(l+m)!}} P_l^m(x),$$

$P_l^m(x)$ are the associated Legendre functions of the first kind,

$$P_l^m(x) := (1-x^2)^{m/2} \frac{d^m P_l(x)}{dx^m}, \quad m \geq 0,$$

and

$$P_l(x) := \frac{(-1)^l}{2^l l!} \frac{d^l}{dx^l} (1-x^2)^l.$$

For $m < 0$

$$\Theta_{lm}(x) = (-1)^m \Theta_{l,-m}(x).$$

Let $h_l(r)$ be the spherical Hankel functions of the first kind, normalized so

that $h_l(kr) \sim e^{ikr}/r$ as $r \rightarrow +\infty$. Let $B_R := \{x : |x| \leq R\} \supset D$, and the origin is inside D .

Then in the region $r > R$, the solution to the acoustic wave problem

(1)-(3) is of the form:

$$u(x, \alpha) = e^{ik\alpha \cdot x} + \sum_{l=0}^{\infty} A_l(\alpha) \psi_l(x), \quad |x| > R,$$

$$\psi_l := Y_l(\alpha') h_l(kr), \quad r > R, \quad \alpha' = x/r,$$

where

$$\sum_{l=0}^{\infty} := \sum_{l=0}^{\infty} \sum_{m=-l}^l .$$

Fix $\epsilon > 0$, an arbitrary small number. The following Lemmas and Theorem1

are proved in [1].

Lemma 1. *There exist $L = L(\epsilon)$ and numbers $c_l = c_l(\epsilon)$ such that*

$$\|u_0(s) + \sum_{l=0}^L c_l(\epsilon) \psi_l(s)\|_{L^2(S)} < \epsilon. \quad (5)$$

Lemma 2. *If (5) holds, then $\|v_\epsilon(x) - v(x)\| = O(\epsilon)$, $\forall x \in D'$, $\epsilon \rightarrow 0$.*

where

$$v_\epsilon(x) := \sum_{l=0}^L c_l(\epsilon) \psi_l(x), \quad x \in D', \quad (6)$$

and

$$\|\cdot\| := \|\cdot\|_{H_{loc}^m(D')} + \|\cdot\|_{L^2(D'; (1+|x|)^{-\gamma})}, \quad \gamma > 0, m > 0, \quad (7)$$

m is arbitrary, and H^m is the Sobolev space.

Lemma 3. $c_l(\epsilon) \rightarrow A_l(\alpha)$, $\forall l, \epsilon \rightarrow 0$.

Theorem 1 (Modified Rayleigh Conjecture). *Let $D \in R^3$ be a bounded obstacle with Lipschitz boundary S . For any $\epsilon > 0$ there exists $L = L(\epsilon)$ and $c_l(\epsilon) = c_{lm}(\epsilon)$, $0 \leq l \leq L$, $-l \leq m \leq l$, such that inequality (5) holds. If (5) holds then function (6) satisfies the estimate $\|v(x) - v_\epsilon(x)\| = O(\epsilon)$, where the norm is defined in (7). Thus, $v_\epsilon(x)$ is an approximation of the scattered field everywhere in D' .*

In order to obtain an accurate solution, usually one has to take L large.

But as L grows the condition number of the matrix $(\psi_l, \psi_{l'})_{L^2(S)}$ is increasing very fast. So we choose some interior points $x_j \in D$, $j = 1, 2, \dots, J$, and use the following version of Theorem 1([2]):

Theorem 2. Suppose $x_j \in D$, $j = 1, 2, \dots, J$, then $\forall \epsilon > 0$, $\exists L = L(\epsilon)$

and $c_{lj}(\epsilon)$, $l = 0, \dots, L$, $j = 0, \dots, J(\epsilon)$, such that

(i)

$$\|u_0(s) + \sum_{j=0}^J \sum_{l=0}^L c_{lj}(\epsilon) \psi_l(s - x_j)\|_{L^2(S)} < \epsilon. \quad (5')$$

(ii)

$$\|v_\epsilon(x) - v(x)\| = O(\epsilon),$$

where

$$v_\epsilon(x) = \sum_{j=0}^J \sum_{l=0}^L c_{lj}(\epsilon) \psi_l(s - x_j)$$

and the $\|\cdot\|$ is defined in Lemma 2.

Remark. Theorem 1 is the basis for MRC algorithm for computation of the field scattered by an obstacle: one takes an $\epsilon > 0$ and an integer $L > 0$, minimizes the left-hand side of (5) with respect to c_l , and if the minimum is $\leq \epsilon$ then the function (6) is the approximate solution of the obstacle scattering problem with the accuracy $O(\epsilon)$. If the above minimum is greater than ϵ , then one increases L until the minimum is less than ϵ . This is possible by Lemma 1. In computational practice, one may increase also the number J of points x_j inside D , as explained in Theorem 2. The increase of J allows one to reach the desired value of the above minimum keeping L relatively small. This gives computational advantage in many

cases.

In section 2, an algorithm is presented for solving the problem (1)-(3).

This algorithm is based on the MRC. Compared with the previous work in the case of two- and three-dimensional MRC([2],[3]), we consider more general surfaces, in particular non-starshaped and piecewise-smooth boundaries. The numerical results are given in section 3. A discussion of the numerical results is given in section 4.

II. The MRC algorithm for Solving Obstacle Scattering Problems

1. Smooth starshaped boundary:

Assume the surface S is given by the equation

$$r = r(\theta, \varphi), \quad 0 \leq \varphi \leq 2\pi, \quad 0 \leq \theta \leq \pi.$$

Define

$$F(c_0, c_1, \dots, c_L) := \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S)}^2. \quad (5'')$$

Let

$$h_1 = 2\pi/n_1, \quad h_2 = \pi/n_2$$

$$0 = \varphi_0 < \varphi_1 < \dots < \varphi_{n_1} = 2\pi, \quad \varphi_{i_1} = i_1 h_1, \quad i_1 = 1, \dots, n_1,$$

$$0 = \theta_0 < \theta_1 < \dots < \theta_{n_2} = \pi, \quad \theta_{i_2} = i_2 h_2, \quad i_2 = 1, \dots, n_2,$$

where n_1 and n_2 are the number of steps. By Simpson's formula([8]), we obtain an approximation of $F(c_0, c_1, \dots, c_L)$:

$$F(c_0, c_1, \dots, c_L) = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2}| + \sum_{l=0}^L c_l \psi_{l i_1 i_2} |^2 w_{i_1 i_2} h_1 h_2 \quad (5''')$$

where

$$a_{i_1, i_2} = \begin{cases} 4, & i_1 \text{ and } i_2 \text{ even} \\ 8, & i_1 - i_2 \text{ odd} \\ 16, & i_1 \text{ and } i_2 \text{ odd} \end{cases}$$

and

$$\psi_{l i_1 i_2} = Y_l(\theta_{i_1}, \varphi_{i_2}) h_l(kr(\theta_{i_1}, \varphi_{i_2})), \quad w_{i_1 i_2} = w(\theta_{i_1}, \varphi_{i_2})$$

where

$$w(\theta, \varphi) = (r^2 r_\varphi^2 + r^2 r_\theta^2 \sin^2 \theta + r^4 \sin^2 \theta)^{1/2}. \quad (8)$$

We can find $c^* = (c_0^*, c_1^*, \dots, c_L^*)$ such that

$$F(c^*) = \min F(c_0, c_1, \dots, c_L). \quad (9)$$

We first write

$$F(c) = \|Ac - B\|^2, \quad (10)$$

where

$$A = (A_{l,i})_{M \times L_1}, \quad A_{l,i} = \psi_{l i_1 i_2} (a_{i_1 i_2} w_{i_1 i_2} h_1 h_2)^{\frac{1}{2}}, \quad i = i_1 i_2,$$

$$B = (B_i)_{M \times 1}, \quad B_i = u_{0i_1i_2} (a_{i_1i_2} w_{i_1i_2} h_1 h_2)^{\frac{1}{2}},$$

in which $M = n_1 n_2$, $L_1 = (L + 1)(2L + 1)$ since $c_l = c_{lm}$, $0 \leq l \leq L$, $-l \leq m \leq l$.

Then Householder reflections are used to compute an orthogonal-triangular factorization: $A * P = Q * R$ where P is a permutation([8], p.171), Q is an orthogonal matrix, and R is an upper triangular matrix. Let $r = \text{rank}(A)$. This algorithm requires $4ML_1r - 2r^2(M + L_1) + 4r^3/3$ flops([9], pp.248-250). The least squares solution c is computed by the formula $c = P * (R^{-1} * (Q' * (A^T B)))$. This minimization procedure is based on the matlab code([10]).

In [2] and [3] singular value decomposition was used for minimization of (5"). Here we use the matlab minimization code which is based on a factorization of the matrix A . This has the following advantages from the point of view of numerical analysis. We can choose an integer r_1 :

$$0 < r_1 \leq r$$

such that the first r_1 rows and columns of R form a well-conditioned matrix when A is not of full rank, or the rank of A is in doubt([10]). See Golub and Van Loan [9] for a further discussion of numerical rank determination.

If we choose $x_j \in D$, $j = 1, \dots, J$, we obtain

$$F_J(c) = F_J(c_{01}, \dots, c_{0J}, c_{11}, \dots, c_{1J}, \dots, c_{L1}, \dots, c_{LJ})$$

$$= \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{j=1}^J a_{i_1 i_2} |u_{0 i_1 i_2}| + \sum_{l=0}^L c_l |\psi_{l i_1 i_2}|^2 w_{i_1 i_2} h_1 h_2.$$

The algorithm for finding the minimum of $F_J(c)$ will be same.

2. Piecewise-smooth boundary:

Suppose

$$S = \bigcup_{n=1}^N S_n.$$

Then

$$\begin{aligned} F(c_0, c_1, \dots, c_L) &= \sum_{n=1}^N \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S_n)}^2 \\ \forall (x, y, z) \in S_n, \quad r^2 &= x^2 + y^2 + z^2, \quad \cos \theta = z/r, \quad \tan \varphi = y/x. \end{aligned} \quad (11)$$

3. Non-starshaped case:

Suppose S is a finite union of the surfaces, each of which is starshaped

with respect to a point \vec{r}_n^0 ,

$$S = \bigcup_{n=1}^N S_n.$$

and the the surfaces S_n are given by the equations in local spherical coordinates:

$$S_n : \quad \vec{r} - \vec{r}_n^0 = (r_n(\theta_n, \varphi_n) \cos \varphi_n \sin \theta_n, r_n(\theta_n, \varphi_n) \sin \varphi_n \sin \theta_n, r_n(\theta_n, \varphi_n) \cos \theta_n),$$

$$n = 1, \dots, N,$$

where \vec{r}_n^0 are constant vectors.

Then

$$F(c_0, c_1, \dots, c_L) = \sum_{n=1}^N \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S_n)}^2.$$

The weight functions $w_n(\theta, \varphi)$ are the same as in (8) since \vec{r}_n^0 are constant vectors.

III. Numerical Results

In this section, we give four examples to show the convergence rate of the algorithm and how the error depends on the shape of S .

Example 1. The boundary S is the sphere of radius 1 centered at the origin.

In this example, the exact coefficients are:

$$c_{lm} = -\frac{4\pi i^l j_l(k)}{h_l(k)} \overline{Y_{lm}(\alpha)}$$

Let $k = 1$, $\alpha = (1, 0, 0)$. We choose $n_1 = 20$, $n_2 = 10$.

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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F(c^*)	6.3219	1.6547	0.2785	0.0368	0.0034	0.0003	0.0000	0.0000
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err(c)	0.0303	0.0172	0.0020	0.0004	0.0000	0.0000	0.0000	0.0000
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where

$$err(c) = \left(\sum_{l=0}^L |c_l^* - c_l|^2 \right)^{\frac{1}{2}}.$$

When $n_1 = 40$, $n_2 = 20$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3544	1.6562	0.2820	0.0358	0.0036	0.0003	0.0000	0.0000
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$err(c)$	0.0147	0.0076	0.0011	0.0001	0.0000	0.0000	0.0000	0.0000
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Next, we fix $n_1 = 20$, $n_2 = 10$ and test the results for different k and α .

When $k = 2$, $\alpha = (1, 0, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	10.4506	5.5783	1.9291	0.5217	0.0970	0.0156	0.0020	0.0003
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$err(c)$	0.0404	0.0205	0.0048	0.0020	0.0005	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (0, 1, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3801	1.6628	0.2821	0.0371	0.0044	0.0003	0.0000	0.0000
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err(c)	0.0014	0.0106	0.0005	0.0004	0.0000	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (0, 0, 1)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.4156	1.6909	0.2955	0.0418	0.0025	0.0002	0.0000	0.0000
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err(c)	0.0093	0.0109	0.0049	0.0007	0.0001	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (1/\sqrt{2}, 1/\sqrt{2}, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$ 6.3500 1.6711 0.2810 0.0371 0.0040 0.0003 0.0000 0.0000

$\text{err}(c)$ 0.0218 0.0057 0.0019 0.0004 0.0001 0.0000 0.0000 0.0000

When $k = 1$, $\alpha = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$,

L 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000

$F(c^*)$ 6.3739 1.6542 0.2850 0.0368 0.0040 0.0003 0.0000 0.0000

$\text{err}(c)$ 0.0170 0.0054 0.0021 0.0003 0.0001 0.0000 0.0000 0.0000

Example 2. The boundary S is the surface of the cube $[-1, 1]^3$. Here

$$S = \bigcup_{n=1}^6 S_n.$$

and

$$\begin{aligned} F(c_0, c_1, \dots, c_L) &= \sum_{n=1}^6 \left\| u_0 + \sum_{l=0}^L c_l \psi_l \right\|_{L^2(S_n)}^2 \\ &= \sum_{n=1}^6 \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2}|^2 + \sum_{l=0}^L c_l \psi_{l i_1 i_2} |^2 \Delta_1 \Delta_2 \end{aligned}$$

where

$$\Delta_1 = 2/n_1, \quad \Delta_2 = 2/n_2.$$

The origin is chosen at the center of symmetry of the cube. The surface area element is calculated in the Cartesian coordinates, so the weight $w = 1$.

Let S_1 be the surface

$$z = 1, \quad -1 \leq x \leq 1, \quad -1 \leq y \leq 1$$

and

$$x_{i_1} = -1 + i_1 \Delta_1, \quad 0 \leq i_1 \leq n_1$$

$$y_{i_2} = -1 + i_2 \Delta_2, \quad 0 \leq i_2 \leq n_2$$

Then

$$\psi_{l i_1 i_2} = Y_l(\theta_{i_1}, \varphi_{i_2}) h_l(kr(\theta_{i_1}, \varphi_{i_2})),$$

and θ_{i_1} and φ_{i_2} can be computed by formula (11). For other surfaces S_j the algorithm is similar.

The values of $\min F(c) = F(c*)$ and the values $\min F_J(c) = F_J(c*)$ with

x_j :

$$\{x_j : j = 0, \dots, 6\} = \{(0, 0, 0), (0.2, 0, 0), (-0.2, 0, 0),$$

$$(0, 0.2, 0), (0, -0.2, 0), (0, 0, 0.2), (0, 0, -0.2)\}$$

are given below.

We choose $n_1 = 10, n_2 = 10$

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
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$F(c^*)$	10.6301	3.6277	2.6760	2.2309	1.9832	1.5737	1.5034	1.2948	1.1753
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$F_J(c^*)$	2.6297	1.0970	0.5487	0.1572	0.0667	0.0320	0.0168	0.0078	0.0035
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When $n_1 = 20$, $n_2 = 20$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
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$F(c^*)$	10.7923	3.7144	2.7778	2.3393	2.0873	1.6671	1.5938	1.4277	1.3368
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$F_J(c^*)$	2.7248	1.1433	0.5757	0.1686	0.0694	0.0652	0.0236	0.0143	0.0090
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Example 3. The boundary S is the surface of the ellipsoid $x^2 + y^2 + z^2/b^2 = 1$, the values of $\min F(c) = F(c^*)$, $b = 2, 3, 4, 5$ with $n_1 = 20$, $n_2 = 10$ are:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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b=2 8.8836 5.4955 3.0421 2.8434 1.3622 1.2093 0.8753 0.8132

b=3 14.1617 12.0477 7.2296 7.0999 3.8077 3.6829 3.1324 3.0496

b=4 19.5326 17.9346 9.9927 9.8720 5.3333 5.2008 4.6793 4.5738

b=5 22.9765 21.5653 11.4850 11.3587 6.1637 6.0096 5.5202 5.3933

The values of $\min F_J(c) = F_J(c*)$, $b = 2, 3, 4, 5$ with x_j :

$$\{x_j : j = 0, \dots, 6\} = \{(0, 0, 0), (0.5, 0, 0), (-0.5, 0, 0),$$

$$(0, 0.5, 0), (0, -0.5, 0), (0, 0, 0.5), (0, 0, -0.5)\}$$

are:

L 0 1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000

b=2 2.4856 0.7090 0.2530 0.0062 0.0000 0.0000 0.0000 0.0000

b=3 4.6639 1.3619 0.6618 0.0074 0.0000 0.0000 0.0000 0.0000

b=4 5.5183 1.8624 0.7844 0.0060 0.0000 0.0000 0.0000 0.0000

b=5 11.0579 8.7027 6.4831 0.8357 0.0017 0.0000 0.0000 0.0000

Example 4. The obstacle is a dumbbell. Its boundary S is not smooth, non-starshaped and not convex:

$$S = S_1 \cup S_2 \cup S_3$$

$$S_1 : \vec{r} - (0, 0, 1) = (1.5 \cos \varphi \sin \theta, 1.5 \sin \varphi \sin \theta, 1.5 \cos \theta)$$

$$S_2 : \vec{r} - (0, 0, -1) = (1.5 \cos \varphi \sin \theta, 1.5 \sin \varphi \sin \theta, 1.5 \cos \theta)$$

$$S_3 : r \sin \theta = 1$$

$$\{x_j : j = 0, \dots, 10\} = \{(0, 0, 0), (0, 0, 0.1), (0, 0, -0.1), (0, 0, 0.2), (0, 0, -0.2),$$

$$(0, 0, 0.3), (0, 0, -0.3), (0, 0, 0.4), (0, 0, -0.4), (0, 0, 0.5), (0, 0, -0.5)\};$$

We choose $n_1 = 20$, $n_2 = 10$ for every $S_i (i = 1, 2, 3)$.

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	25.8840	20.8059	16.4968	15.6622	12.9241	12.1915	11.0187	9.5263
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$F_J(c^*)$	20.3118	8.0238	5.1062	2.5908	0.8304	0.4067	0.0453	0.0084

IV. Conclusion

From the numerical results one can see that the accuracy of the numerical solution depends on the smoothness and elongation of the object.

In Example 1 the surface S is a unit sphere and the numerical solution is very accurate. In Example 3 the results for different elongated ellipsoids show that if the elongation (eccentricity) grows, then the accuracy decreases. In Example 2 the surface is not smooth and the result is less accurate than in Example 3. In Example 4 the surface is nonconvex and not smooth, but the accuracy is of the same order as in Example 2.

When b is large or S is not smooth, the numerical results in Example 2 and Example 3 show that if one adds more points x_j then the accuracy of the solution increases.

In Example 1 and Example 2, as one increased n_1 and n_2 , the minimum

$F(c^*)$ has also increased because the condition number of the matrix A in (10) grew as n_1 and n_2 increased.

Using the results of Example 1 one can check the accuracy in finding c_l by the value of the minimum

$$F(c^*) \leq \epsilon.$$

References

- [1] Ramm A. G. [2002], Modified Rayleigh Conjecture and Applications, J. Phys. A: Math. Gen. 35, L357-L361.
- [2] Gutman S. and Ramm A. G. [2002], Numerical Implementation of the MRC Method for Obstacle Scattering Problems, J. Phys. A: Math. Gen. 35, L8065-L8074.
- [3] Gutman S. and Ramm A. G., Modified Rayleigh Conjecture Method for Multidimensional Obstacle Scattering Problems(submitted).
- [4] Barantsev, R., Concerning the Rayleigh hypothesis in the problem of scattering from finite bodies of arbitrary shapes, Vestnik Lenigrad. Univ., Math., Mech., Astron., 7, (1971), 52-62.
- [5] Millar, R., The Rayleigh hypothesis and a related least-squares solution to scattering problems for periodic surfaces and other scatterers, Radio, Sci., 8, (1973), 785-796.
- [6] Ramm, A. G., Scattering by obstacles, D. Reidel, 1986.
- [7] Triebel H., Theory of Function Spaces, vol. 78 of Monographs in Mathematics. Birkhauser Verlag, Basel, 1983.
- [8] Kincaid D. and Cheney W., Numerical Analysis: Mathematics of Scientific Computing, Brooks/Cole, 2002.
- [9] Golub G. H. and Van Loan C. F., Matrix Computations, The John

Hopkins University Press: Baltimore and London, 1996.

- [10] Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Don-
garra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D.
Sorensen, LAPACK User's Guide (http://www.netlib.org/lapack/lug/lapack_lug.html),
Third Edition, SIAM, Philadelphia, 1999.